a generalization of those of Chebyshev), and they provide the solution to an unusual number of extremal problems. They also play an important role in interpolation theory and numerical integration. Less well known are properties of these polynomials which have a basis in modern analysis, especially ergodic theory, as well as in algebra and number theory.

The book under review not only supplies the important results concerning Chebyshev polynomials in these areas, but also contains a basic introduction to approximation and interpolation theory. Written in a lucid style, which illuminates the beautiful topics covered, the author has enhanced his work by the inclusion of over 300 interesting exercises. As a result, the book can easily be used as a text, especially in a seminar, but it also should be read by anyone wishing to learn about this fascinating subject.

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11[41A99].—PAUL NEVAI & ALLAN PINKUS (Editors), *Progress in Approximation Theory*, Academic Press, Boston, 1991, xi + 916 pp., 23 $\frac{1}{2}$ cm. Price $189.00.

This is a collection of 62 research papers that have been submitted to, and accepted by, the *Journal of Approximation Theory* and, with the authors' permission, have been assembled in this volume in order to alleviate the current backlog of the journal. Accordingly, a great variety of topics, both in pure and applied approximation theory, are being addressed, and the ordering of the papers alphabetically with respect to authors only accentuates this diversity. In character, the papers range from a short 3-page note to a substantial 74-page memoir. The printing conforms exactly to that of the journal, except that no received dates are given.

W. G.

12[68Q40, 65Y15, 65Y25, 11-04, 12-04, 13-04, 14-04, 30-04, 33-04].—STEPHEN WOLFRAM, *Mathematica—A System for Doing Mathematics by Computer*, 2nd ed., Addison-Wesley, Redwood City, California, 1991, xxiv + 961 pp., 23 cm. Price $48.50 hardcover, $33.50 paperback.

Mathematica is an interactive computer software system and language intended for solving problems in mathematics. Prominent features are numerical and symbolic mathematical manipulation, elaborate plotting software using the PostScript display technology, and a versatile programming language. While few of the features are entirely novel or "state of the art," the combination of all of these facilities in an accessible package has made it popular.

Any serious user of Mathematica (version 2.0, corresponding to the system described in this publication) will probably wish to have this book close at hand. A purchaser of the software will presumably already own one copy of this manual. It is the principal reference for the commands, and the on-line help is not an adequate substitute. Furthermore, a user will most likely continue to need occasional assistance in the use of this complex program.

For those who have no access to the program, this volume is effective as a convincing sales tool for the program: it includes dramatic color plots, numerous examples of interesting and clever mathematical manipulations, and few intimations of any limitations on its capabilities.

In nearly 1000 pages, this text includes a brief "Tour of Mathematica," a "Graphics Gallery" (26 pages of color), and four major sections: a practical introduction, a more detailed description of the features of the system and language, "advanced mathematics," and a reference section. This last portion is a 150-page alphabetical list of nearly all the mathematical functions, language components, and system commands. There is an excellent 50-page index.

The text has been produced through computer typesetting, and benefits from the integration of Mathematica with the PostScript picture-description language. The apparently effortless inclusion of plots, computer input and output, and mathematical text is a model for how such material should be presented.

Although it is not possible in a brief review to summarize the material in this text, some general critical comments seem to be in order:

The disparity in the level of treatment of issues may be distressing to the mathematician. For example, over 100 pages are devoted to graphics, and there are over 50 options to the "Plot3D" command. Yet the "definition" of the natural logarithm is given on one line, and neglects to mention the treatment or even existence of a branch cut in the complex plane. Does this mean the treatment is so obvious and correct that it needs no explanation? Not really, since (for example) one cannot deduce from the documentation the meaning Mathematica would assign to the function $f(z) := \log(z^3) - 3\log(z)$. Nor can one deduce Mathematica's answer if one were to ask it to solve the equation $f(z) = 0$ for z. The current version claims there is no solution, although $-\pi/3 < \arg z \leq \pi/3$ would be more correct.

Other aspects of the program that are hardly documented but can substantially compromise the correctness of computational results include significance arithmetic in the numerical model, strange treatment of asymptotic order, limits and infinities, interval arithmetic, and inequalities. In fact, a subtle implication of the design seems to be that it is acceptable for exact algebraic computation to sometimes produce wrong answers, much as though "experimental error" played a role in arithmetic or algebra.

Purchasers of this book may have various expectations about its contents. In order to avoid disappointment, I am indicating some of the areas **not** covered:

(a) There is no survey, brief or otherwise, of the fields of computer algebra, numerical analysis, graphics, or programming. The author provides bibliographic citations only for books using Mathematica, not source material for understanding the technology. This seems to reinforce the author's viewpoint that Mathematica is a new and unique "black box" for "doing mathematics."

Readers of this review may be aware of a substantial literature on numerical computation as well as graphics, although references on symbolic computation (for example [2], [4]) may be less well known.

(b) There are no descriptions of, or references for, the algorithms that are used. There is virtually no description of how the system performs any of its mathematical tasks. Knowledge of the details of these algorithms, from arbitrary-precision numerical evaluation of special functions, to symbolic definite integration, is of interest for many reasons. Even within the goal of providing information to users of the program, such information would make it possible to avoid some of the flaws in the methods. Admittedly, the kind of extensive details given in Buchberger et al. [2] would be out of place in a mere users' manual, but this same volume is also the system's only reference document. By comparison, Maple's manual [3] indicates the algorithms used and has much more detailed information and references. Maple goes further by providing much of its source code.

(c) There is insufficient material to provide a detailed understanding of the program. The welter of detail on system "global flags" and special commands such as Together, Expand, ComplexExpand, PowerExpand, etc. should not obscure the fact that about the only way to tell if an utterance in the language is syntactically or semantically meaningful is to type it into the system and (in some cases) check the results by some independent means.

(d) There is insufficient information to provide an explanation of what parts of mathematics are "known" to the system. There are numerous "built-in" mathematical functions from the natural logarithm to generalized hypergeometric functions. In some cases the system can do no more than evaluate the function at a numeric point. Some functions can be differentiated. Some can appear in integrands. Some can be rewritten in terms of other functions or simplified in various ways. Usually, such implicit transformations are undocumented, and implicit or explicit transformations may make unwarranted range assumptions so as to be, in fact, incorrect. Although there are many instances in which Mathematica gives wrong answers, Wolfram admits such a possibility on page 89, only with respect to definite integrals: "... Mathematica may give results, which, while formally correct, will lead to incorrect answers when you substitute particular values for variables." One need not be devious to find problems. In a problem recently given to calculus students here, Mathematica failed to find the arc length of the parametric curve $c(t) := (t - \sin t, 1 - \cos t)$ from $t = 0$ to 2π.

(e) There is an insufficiently precise description of the programming language. It is nearly self-evident that it is important to have a clear description of an algorithmic language. This is vital in Mathematica because many users inevitably find it necessary to build up collections of procedure definitions to augment the built-in interactive commands. Since the language draws from many previous languages, several equivalent programming "paradigms" coexist. The choice of the wrong technique can make a program quite slow. In fact the favored technique of using patterns and rules is appealing in simple cases, but it can cause subtle errors, inconsistencies and gross inefficiencies.

A computer scientist trying to understand the language may be hindered by the omission of a formal definition of the syntax in Backus-Naur Form (BNF). Wolfram's treatment of modules, blocks, contexts, and packages is confusing, but perhaps no more so than is required by the apparently confused design of scopes, evaluation, and variables. For example, the system does not understand that the scope of the x in $\int f(x)\,dx$ is local to the integral.

Other books, especially those by Maeder [6] and Blachman [1] provide alternative perspectives on the language and point out some traps and pitfalls for the unwary. A simple such example is that no distinction is made between a column-vector and a row-vector.

Although we have tried to restrict our comments here to the book under review, readers may wish to refer to reviews of the Mathematica *program* as well. Simon [7] compares Mathematica to several other systems in terms of computational speed and convenience on a set of problems. Another review [5] discusses at length how well the program fulfills the goals set out by the designers.

Readers who have become aware, through Mathematica, of the general capabilities of symbolic manipulation programs may find it beneficial to examine literature on alternative systems such as Derive, Macsyma, Maple, Reduce, and Theorist. Determining the "best" system is necessarily dependent on individual circumstances. At the very least, it appears that some of these other systems get correct answers when Mathematica does not. Quoting further specific flaws or discrepancies between the documentation and the system may be pointless because the behavior of the system may be changed freely.

What we have observed is consistent with the disclaimer on the inside title page: "The author, Wolfram Research and Addison-Wesley shall not be responsible under any circumstances for providing information on or corrections to errors and omissions discovered at any time in this book or the software it describes, whether or not they are aware of the errors or omissions. The author, Wolfram Research and Addison-Wesley do not recommend the use of the software described in this book for applications in which errors or omissions could threaten life, injury or significant loss."

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13[68-06].—DAVID V. CHUDNOVSKY & RICHARD D. JENKS (Editors), *Computers in Mathematics*, Lecture Notes in Pure and Appl. Math., Vol. 125, Marcel Dekker, New York and Basel, 1990, vii + 410 pp., 25 cm. Price $89.75.

This volume contains 14 contributions to the International Conference on Computers and Mathematics, which took place July 29–August 1, 1986, at Stanford University. The papers deal with the role of computers in subjects